

A WRITING INTENSIVE STATISTICS COURSE

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ABSTRACT: We discuss an upper division applied statistics course that has been structured to satisfy our college's writing intensive requirements within the mathematics major. In this course, students complete two projects, performing both a statistical survey and a controlled experiment, and then write up their work on each project in a formal paper. After each paper, the students read a group of papers written by their peers and then critique and rank these papers. Next, the students meet in groups, having read the same papers, and together they must form a consensus about their group rankings. Finally, after analyzing and discussing several papers from their peers, the students revise their own papers.

KEYWORDS: Writing, statistics, projects.

THE WRITING INTENSIVE REQUIREMENT

Many colleges and universities have implemented writing intensive requirements for students of all majors. Carroll College is a small liberal arts institution with 1500 students in Helena, Montana, and it has recently implemented this type of requirement, demanding that, in addition to the usual college composition course, all students must take two courses designated as writing intensive, one within their major and one in another discipline. In order for a course to be designated as writing intensive, the curriculum committee must approve that the course meets the following criteria:

- The course must provide systematic instruction in writing; that is, there must be more than one class period devoted to writing instruction, where teachers discuss the criteria of writing in their discipline.

- The course must give attention to the process of writing; that is, students should get the opportunity to turn in written work with the opportunity to improve through revision or application of learned concepts.
- The course must require students to turn in and receive feedback on a series of writing assignments during the semester.

The writing intensive mathematics course is just one of several important writing courses that our students must take. All students must take EN 102 College Composition to learn the fundamentals. For our mathematics major we also require EN 325 Technical Writing, which teaches writing specifically for the sciences and technology.

THE COURSE AND CONTEXT

At Carroll College, we have a small mathematics major focused on applied mathematics, with generally about a dozen students in our upper division courses. We place emphasis on the ability to communicate mathematics throughout our curriculum. In each of our lower division courses for mathematics and engineering students, we include a small group term project, the results of which must be written up in a formal paper as well as presented to the class with a 5-10 minute PowerPoint presentation. Further we use the “Good Problems” process in our freshman calculus course [1]. Approximately once each week, we take a few minutes to discuss one of the fundamental components of mathematical writing, and accompany this with a handout that gives both good and bad examples. These components include “Introductions and Conclusions,” “Flow,” and “Graphing.” At the same time we designate one of the homework problems as “Good Problem” and require the students to carefully write up this problem on a separate sheet of paper, using complete sentences to explain each step of their work. This problem is then graded separately, based as much on writing and presentation as mathematical correctness, and the problem counts as much as an entire regular homework assignment.

Given our general emphasis on mathematical communication, the creation of an upper division writing intensive course fits well into our curriculum. We selected “Math 341: Probability and Statistics II” to become our writing intensive course. This three-credit course is generally taken in the fall of our students’ junior year and is a follow-up to our two-credit “Math 336 Probability and Statistics I.” The first course emphasizes probability, with a bit of linear regression, while Math 341 covers joint probability and a

variety of topics in applied statistics, including confidence intervals, hypothesis testing, and multiple regression. As a text, we use “Applied Statistics and Probability for Engineers” by Montgomery and Runger [2]. In keeping with the focus of our curriculum, this course emphasizes applied statistical methods, the mathematical underpinnings of these methods, and how they can be used to analyze a variety of real-world situations. Thus this course is a calculus-based statistical methods course, rather than a “math-stats” course emphasizing formal derivations of these methods and proofs of the relevant theorems.

GENERAL STRATEGY

The point of good mathematical writing is to write simply and plainly, to take a complex idea and explain it with clarity. Good mathematical writing should present an argument so that the reader can easily follow the logic of each step as we work towards the conclusion. Equations should be discussed and explained. Graphs and plots should illustrate key points. Good mathematical writing is clear evidence of good mathematical thinking, which is why it is so important.

How do you teach good mathematical writing? One approach would be for the instructor to give writing assignments, to make comments on the student work, and then to have the students revise their papers accordingly. However, by the time our students reach this course, they have been through the process of instructor-directed revision more than once in other courses. It would be more useful if we could teach our students to criticize their own writing, to read their own papers with an analytical eye, to see flaws and difficulties and then to revise their work accordingly. To achieve this more ambitious goal, we begin with the observation that it is easier to see common mistakes and problems in the writing of others than in one's own writing. So we could ask students to read and critique papers written by their peers. However, when put in this position, most students will do almost anything to avoid making negative comments about the work of their peers. Instead they will blandly approve of everything, and criticize only the most minor technical points. We must construct this assignment so that the students cannot avoid identifying significant flaws in the work of other students. Thus, we require the students to read a group of papers, and to numerically rank them from best to worst. They must then write a paragraph about each paper, explaining and defending each ranking. This forces the student to rank one paper last and to explain this ranking by pointing out its flaws. After our students have gone through this exercise,

requiring them to seriously criticize the work of their peers, we ask them to turn to their own writing and to revise it, applying what they have learned from this process. Using this general approach, we design the assignments to teach the process of mathematical writing and revision.

ASSIGNMENT #1: ANALYZE PAPERS FROM LAST YEAR'S CLASS

We begin the course by spending the first two weeks developing a conceptual understanding of sampling distributions, confidence intervals, hypothesis testing, and p -values. These are the central ideas of the course that will appear again and again in a variety of contexts, so we devote this time to building a solid foundation, and dealing with common misconceptions, so that students can work towards a clear idea of what these mean, and what they do not mean. We learn how to calculate confidence intervals and p -values in the simplest context, focusing on the mean of a normal population with known variance, so that the computational procedure is quite simple, allowing us to deal with the more fundamental issues.

In the third week, as the course moves on to other topics, we hand out four of the better papers written by the previous year's class: two statistical surveys and two controlled experiments. The students then have two weeks to read and analyze these papers, to rank them from best to worst, and to write a short paper of their own with a one-paragraph critique of each paper (see Appendix for assignment handout). While the students have not yet studied the mathematics used in most of these papers, they have the basic conceptual ideas, and a good paper should explain the mathematics in enough detail to be comprehensible.

This assignment not only introduces the students to the process of reading a mathematical paper with a critical eye, it also gives the students a clear idea of what is involved in a term project, so that they know where to start when designing their own projects. The biggest criterion in grading these critiques is to make sure that the students are considering these papers in depth, and pointing out substantial issues, rather than just mentioning technical issues, spelling or punctuation errors.

ASSIGNMENT #2: A STATISTICAL SURVEY

The students are asked to perform a statistical survey of a large population, to analyze the results of their survey using the tools of the course, creating confidence intervals or performing hypothesis tests, and to carefully write

up their work in a formal paper (see Appendix for assignment handout). No other instructions are given as to the topic of the project, so the range of subjects is very diverse. I have had students study the distribution of colors of M&M candies, the ratio of bran to marshmallows in Lucky Charms cereal, the speeds of different web browsers, the results of football games, the prices of items in different grocery stores, and student opinions about parking on campus. Doing this sort of study gives the students an appreciation for the challenges of real statistical work, and the difficulties of gathering a truly random sample to eliminate sources of bias in the sample.

The students write their papers using next year's class as the target audience, so the paper must explain the calculations in enough detail so that students with only a basic conceptual understanding can understand what has been done. They make a 5-10 minute PowerPoint presentation to the class, and at this point they receive a grade mostly based on completion. Every student who puts in the time and effort to complete a meaningful project on time receives the points.

Next, we begin the peer review process: The class is broken into groups of 4 or 5, and each group provides another group with copies of their papers, so that everyone in group A reads all the papers written by group B, group B reads the papers from group C, and so forth. Each student is given a week to read the papers, to rank them from strongest to weakest, and to write a paragraph on each paper explaining its ranking. In a class of this size, it is not possible to make the review process blind, so students are simply reminded to be professional and objective in this assignment, and no problems have arisen so far. To encourage students to make serious criticisms, they are reassured that the original authors of the papers will never see their reviews, and that the peer reviews will have no effect on grades.

The ranking of papers is graded carefully, focused on two main issues: First, the criticisms should be substantial, focusing on mathematics and clarity rather than technical issues like spelling and punctuation. Second, the criticisms must be specific, referring to particular equations, paragraphs, or even sentences. Rather than just saying "this is unclear" the paper must point out exactly where things are confusing or in error, listing the pages and paragraphs in question.

Some students may worry about how their own paper has been ranked, so it can be helpful at this point to emphasize again that the ranking has nothing to do with the grades that will be assigned: The purpose of the ranking assignment is to learn how to criticize a paper, as well as to learn from the successes and failures that can be seen in peer work. All grades

are assigned by the instructor.

On the day that these rankings are due, we break up into groups, sending groups off to different rooms, and have them spend most of a 50-minute class period comparing their rankings, discussing their differences, agreeing on a group ranking, and filling out a handout explaining their group's thinking (see Appendix). If a student does not turn in their rankings, then they are not permitted to participate in the discussion, so they lose the points.

The written ranking paper demands that every student form opinions about the relative merits of each project. This means that they all begin the discussion with strong feelings about what the issues are, which turns out to be very effective at motivating the resulting discussion. This process of comparing and analyzing the papers as a group is remarkably fruitful, with the students probing the mathematical concepts quite deeply, ferreting out the problems and biases that inevitably arise. The students learn a great deal from each other in this discussion, as they point out the different issues that some may have missed. Each group turns in one handout, and points are assigned equally to all students in the review group.

After the discussion day, students are given a week to go back through their own paper and revise it. Further, they must also turn in a brief statement explaining the changes that they made in revision, permitting assessment of the revision process. The final draft is the one that is carefully graded and comprises the bulk of the points for the project. This is generally the first time that students have been asked to revise their work without first receiving feedback from a teacher, and this will cause anxiety for some. It is important to be completely up-front about the purpose of the assignment, so that students understand that a key point of the exercise is to learn how to critique one's own paper and make revisions independently. Occasionally students have asked to see the rankings and comments that their peers have made, as a guide to revising their own papers. We tried this one year, and it was not very productive. While the student analyses are supposed to be as specific as possible, few are detailed enough to meaningfully guide a revision, and some make comments that are particularly unhelpful.

ASSIGNMENT #3 A CONTROLLED EXPERIMENT

In the second term project each student must perform a controlled experiment, gathering a sample, randomly dividing it into at least two groups, treating the groups differently in some way, performing some measurement after the treatment, and using statistical calculations in order to evaluate whether the differing treatments had a significant effect. Going through

this process and seeing this logic play out helps the students see why a statistical survey can show relationships between different quantities, but can never demonstrate a cause-effect linkage, and exactly how a controlled experiment can make this determination. Designing a true controlled experiment is particularly difficult for many students, requiring a much deeper understanding of the underlying logic than just a survey. For example, one student wanted to study whether visualization could help people perform free-throws in basketball. The student gathered a group of participants and first asked them to attempt ten free-throws. Next each participant was asked to spend 30 seconds visualizing the ball going through the hoop, and try ten more shots. It was only after the project was written up that the student realized that any improvement in scores might simply have come from the practice of the first ten shots.

After the projects are turned in and the presentations made, we again divide the students into groups, have them write up analyses and rankings, spend a class period on discussion, and then assign the students to revise their papers. For this second project, students are placed into different groups, so that they see the widest possible variety of student work and interact with different peers when discussing the rankings.

There tends to be a significant improvement in the quality of the papers between the first and the second project. The more practice that students get at any activity, the more their skills will tend to improve. In particular, the revision of the paper with no instructor feedback, which is intimidating for many students in the first project, is now taken in stride. The students have discovered that they can successfully analyze their own writing and improve it, without a teacher telling them what to do.

GRADING AND TIMELINE

The students receive up to 320 points for all the assignments in these projects (Table 1), making up 32% of each student's final course grade, with the remainder of the grade coming from exams (two midterms and a final) and regular homework assignments. Although the first drafts and the group discussion handouts are graded mostly based on completion, the three rankings and the two revised papers must be carefully graded, which is a considerable commitment of instructor time over the course of the semester. This time commitment is worthwhile because the skills that the students develop are very powerful, and because the time is spent evaluating high-level thinking processes. Relatively few points are assigned for the first drafts of the project and presentations, however students are informed that

if they don't turn in a first draft, then they will not be allowed to turn in a revision, which is a substantial penalty: No first draft, no paper. The presentations generally require a total of four class periods, and the group discussions require the better part of another two class periods, out of a total of 42 50-minute class periods in the term.

Points	Assignment	Dates
20	Rank papers from last year's class	wk 3-4
20	Complete survey paper and presentation	wk 5-7
20	Rank peer survey papers	wk 8
10	Participate in group discussions of survey papers	wk 8
100	Revision of survey papers	wk 9
20	Complete experiment paper and presentation	wk 10-12
20	Rank peer experiment papers	wk 13
10	Participate in group discussions of experiment papers	wk 13
100	Revision of experiment papers	wk 14

Table 1. Assignment points and dates.

CONCLUSIONS

Students must be taught not only how to do mathematics, but how to communicate it. Mathematical writing is an essential skill for any mathematician, whether in a government, private sector, or academic career. Our students can derive considerable benefit if we spend the time to explicitly teach mathematical writing at the undergraduate level. This will give them a head start in graduate school or whatever career they choose to pursue. In designing this course to satisfy the writing intensive requirement for the mathematics major, we view this as the final writing course that most students will take. Thus the motivating question is: Where should our students be in their mathematical writing skills before they graduate? Our students should be able to write up mathematical work in a clear, logical way, and then to revise and improve their work without the external supervision of a teacher telling them what to do. In order to do this, students must be able to look at their mathematical writing with a critical eye, identifying flaws and difficulties themselves. To teach them to critically analyze mathematical writing, we first have our students analyze the writing of their peers, using the requirement of ranking to force them to be critical and look for significant flaws. The purpose of the peer discussions is to show them how others criticized the same papers. Then, after practicing on the papers of their peers, we require our students to look back at their own papers, and to revise them, without explicit guidance from the teacher. By completing

this set of assignments, we can thus give our students considerable practice in the skill of mathematical writing and revision.

REFERENCES

1. Mohlenkamp, M. J., D. Bundy, E. Gibney, J. Mccoll, K. Sandberg, B. Silverstein, B., P. L. Staab, and M. Tearle. 2001. *Good Problems: Teaching Mathematical Writing*. University of Colorado APPM preprint #466, August 15, 2001; <http://amath.colorado.edu/activities/preprints/>.
2. Montgomery, Douglas C. and George C. Runger. 2002. *Applied Statistics and Probability for Engineers*, 3rd ed. New York: Wiley.

APPENDIX: ASSIGNMENT INSTRUCTIONS

Paper Reviews

In this paper (typed, double spaced, written at a college level, your name/date/title on page one), you will analyze several of the projects completed for this class. You will write at least one paragraph about each project, one at a time, ranking the papers (1, 2, 3, etc.) from strongest to weakest. You will base your analysis on the following criteria:

- Accuracy: Are there any mathematical errors in the paper? Is everything correct? Are all statements true?
- Clarity: Does the paper read well? Can you understand it easily? Are there paragraphs that are hard to figure out?
- Logic: Do the data and the calculations support the conclusions drawn by the author?
- Organization: Is the paper properly organized or is it jumbled?
- Scope: Is there more that that could have been done with the data? Are there other things that the author did not think to do or to calculate?
- Figures and Tables: Are they clear? Do they make sense? Do they show patterns fairly obviously or do you have to stare at them and figure them out?

Please be polite and constructive in your criticism. If possible, make suggestions for how the papers could be improved and how problems could be fixed. On the other hand, your analysis must explain and justify your

ranking. So if you rank a paper last, you must state your reasons for giving it such a poor ranking. Don't be afraid to be seriously critical: Your reviews and rankings will have no impact on anyone's grade, and the original author will not be given copies of these rankings. In these rankings do not compare the papers with each other. Instead take the papers one at a time, pointing out the strengths and weaknesses of each paper in turn. More than anything else, *be specific* with both your compliments and your criticism. Nothing is more useless than a vague criticism like "overall things probably should have been more clear." Instead say things like "The first paragraph of the results section is very difficult to understand."

Project 1: A Statistical Survey

The purpose of your project is to do a statistical survey: You will gather a random sample of some large population and analyze it using the methods we have studied in this course. Be very clear about what population you are sampling from, and thus what population your results will apply to. Exactly how a sample of people is gathered is very important in assessing its meaning and applicability. So, be very detailed in describing the selection of your sample. If there are potential biases in how your sample is selected, state them clearly and discuss them

Project 2: A Controlled Experiment

Statistical surveys are very useful, but they cannot determine cause and effect. They can show that two things are related, like smoking and lung cancer, but a survey alone cannot show that one causes the other. To demonstrate that one thing causes another, you must do a controlled experiment: You will gather a random sample from some large population and then randomly divide it into at least two groups, treating the groups differently in some way, performing some measurement after the treatment, and using statistical calculations in order to evaluate whether the differing treatments had a significant effect.

Paper Writing Instructions

All papers must be double spaced, written at a college level, and have your name/date/title on page one. Your paper must provide enough detail that it would allow someone else to duplicate your work and check your results. Your paper must consist of at least the following parts:

- **Summary:** This must be less than a page in length. The summary must briefly explain your entire project, what you did, what you found, and what it means, including specific numerical results (confidence intervals and p -values) as well as your final conclusions. Write the summary last, even though it is the first section of your paper.
- **Introduction:** Explain the background for your project. What are you going to study and why? Be very specific about the purpose of your project: Exactly what question did you try to answer? Exactly what population did you sample?
- **Methods:** Explain how you gathered your data, and justify your methods. Be sure to explain very carefully how your sampling method gathers a random sample from the population that you are investigating. Your paper must present all of your final data, not just means and standard deviations. Every measurement that you use should be included.
- **Results and Analysis:** Present all of the data that you gathered: Tables are often useful for this. Make graphs, charts, or diagrams of your results if at all possible, and go through these figures one by one in the text, explaining what they mean. Describe the calculations that you performed, first giving the general equation from our textbook, citing this equation by number. Explain each step, writing things up carefully like an example in a textbook.
- **Conclusions:** Explain what your study means. Evaluate the strengths and weaknesses of your study. How would you make it better if you had more time?
- **References:** List any books or other resources that you used in this project. For most projects, this is simply our textbook, but reference it formally (giving title, author, year, publishing company, etc.).

Each of these sections should begin with these headings in a large bold font. Within these sections I would suggest using subheadings to further organize things and aid in clarity.

Use cautious and conservative language. State your conclusions tentatively, without making overstatements or broad generalizations. Never say “I feel...” This is a mathematical paper: We are interested in your numbers, your calculations, and your logic, not your feelings. Avoid the word “prove” or “proof.” Statistics deals with probabilities, not absolutes, and in mathematics we reserve the word “prove” for things that are logically certain. (Often the word “test” can be used instead of “prove.”)

Whenever we do statistical hypothesis testing, we are trying to answer the question: Could these results be just due to random variation, or can we rule this out as a cause? Emphasize this! If you do a hypothesis test, calculate a p -value. Be sure to interpret and discuss the meaning of your p -value. In particular, be sure to state exactly what this is the probability of.

Think of figures and tables as containing the evidence that you are using to support the point you are trying to make with your paper. Always remember is that the purpose of a figure or a table is to show a pattern, and when someone looks at the figure this pattern should be obvious. Figures should not be cluttered and confusing; they should make things very clear. Note: If you're going to show me a histogram of continuous data, it must have an adequate number of bins. A histogram of continuous data with only two or three bins cannot show anything useful.

Your target audience for this paper will be next year's class, who will be reading several of our papers after the first few weeks of the term, in the same way you reviewed several of last year's papers. This means that they will have studied basic confidence intervals, p -values, and hypothesis testing, but little more. Remember too, that these papers will be read and evaluated by the other people in this class, so do your best to write them clearly and understandably. The real goal of mathematical writing is to take a complex and intricate subject and to explain it so simply and so plainly that the results are obvious. I want your paper to demonstrate that not only did you do the right calculations, but that you understand what you did and why these methods work.

Please come to me with questions about how to analyze your data. I will be able to point you at the most useful sections of the text, and help you with the process of exactly how to do things. Please check with me to make sure that you are doing things correctly.

Each of you will make a 5-10 minute PowerPoint presentation to the class describing your project. The PowerPoint slides should give the key results of your project and should be detailed enough that I could read them alone and understand what they mean. Any text/numbers in the PowerPoint should be large enough to be easily readable, in at least 28 point font. Graphs and diagrams should be clearly labeled. You should probably spend about one or two minutes talking your way through each transparency, so I wouldn't recommend trying to present more than 7 or 8 slides, and I would suggest that you need at least 3 in order to explain your project.

Committee Reports

1. Compare your rankings with each other and discuss any disagreements. Discuss the strengths and weaknesses of each paper that you read and form a consensus on how to rank each paper. List your committee rankings here:
2. What was the most serious disagreement your group had? Explain both sides of the disagreement.
3. What did your group agree upon the most quickly? Explain why.
4. What was the most common problem that you found in these papers?
5. What advice would you give to next year's class before they begin their term projects?

Now for each of the papers that you read, discuss and form a consensus on both (1) the greatest strength of each paper, and (2) what one change you would recommend to most improve the paper. Please be as specific as possible about both of these.

Project Revisions

Now that you have studied the papers written by your peers, you will revise and improve your term paper. The decision about what to change is yours. You may change anything or leave anything the same, whatever you think will result in a better paper. You may choose to gather more data if you think that would substantially improve things, but the real goal of this revision is to improve the presentation of your data and calculations. Fix errors, rewrite and clarify any points that are vague, remake any figures or tables that are confusing.

In addition to your revised draft, you must write a letter to me listing the changes you have made one by one. Please number these changes so they are easy for me to keep track of, and briefly state why you have made each change, perhaps something like this: "Change 1: I have rewritten the introduction to clarify things. Change 2: I have remade Figure 4 to make the linear trend more obvious. Change 3: I have fixed an error in the first equation on page 7."

BIOGRAPHICAL SKETCH

Originally from Homer, Alaska, Kelly Cline is an assistant professor of mathematics at Carroll College in Helena, Montana. His interests include inno-

vative teaching methods and technologies, including classroom voting and online homework systems.