

Classroom Voting in Linear Algebra

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What is classroom voting?

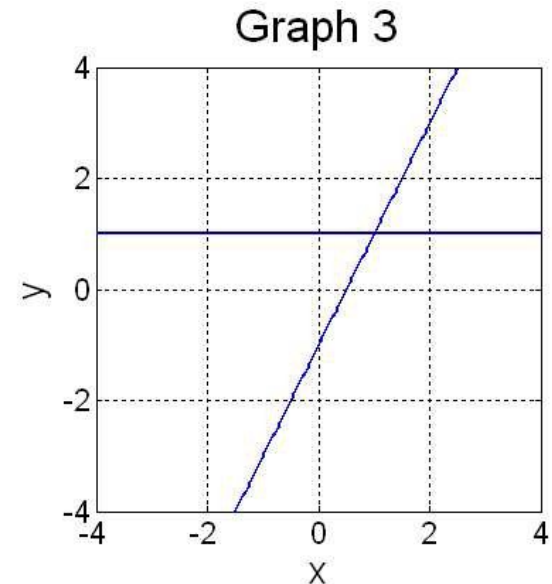
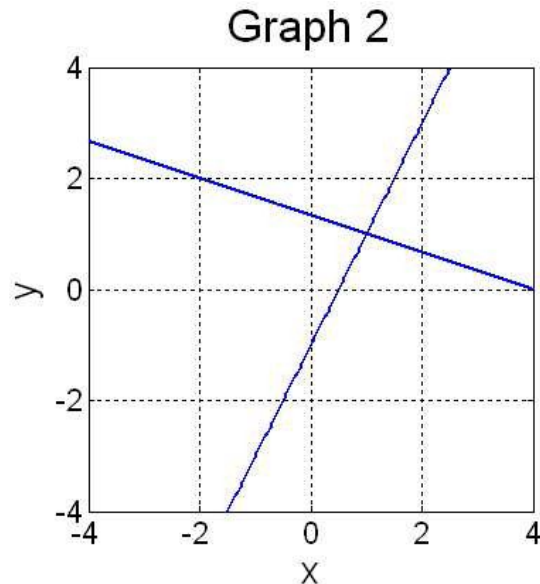
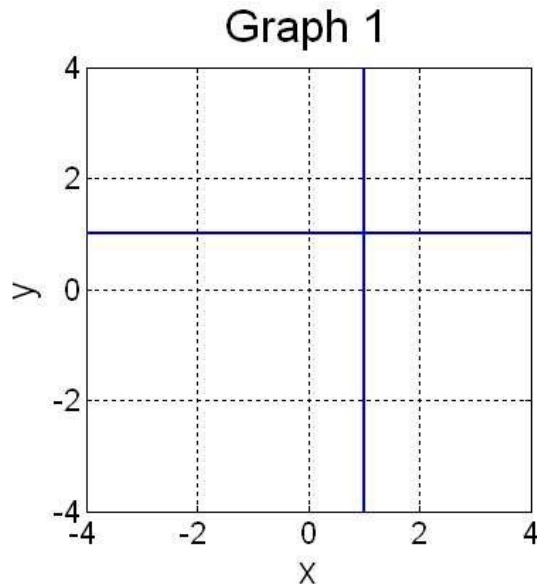
- Present ConcepTests, multiple-choice or true/false questions usually focusing on conceptual issues to the class. Students are given a few minutes to form an opinion and to discuss the question with their neighbors.
- Each student votes on the answer using an electronic clicker or by holding up a colored index card, thus requiring every student to form an opinion.
- The instructor asks individual students to explain and defend their vote.

Gaussian Elimination

- Have done several examples of Gaussian elimination
- Want students to discuss the concept

We have a system of two linear equations and two unknowns which we solve by performing Gaussian elimination on an augmented matrix. Along the way we create the graphs below, showing geometrical representations of the initial system, the system at an intermediate step in the row reduction process, and the system after it has been put into reduced row echelon form.

Put these graphs in order, starting with the initial system and ending with the system in reduced row echelon form.



- (a) Graph 2, Graph 3, Graph 1
- (b) Graph 1, Graph 3, Graph 2
- (c) Graph 1, Graph 2, Graph 3
- (d) Graph 2, Graph 1, Graph 3
- (e) Graph 3, Graph 2, Graph 1

Results of Classroom Voting: Students Get Engaged

- Must take an active role
 - Discuss ideas with peers
 - Think and participate
- Commit to an answer
- Confront common misconceptions during class
- Students have fun! Attendance improves!

Our Experience: Discussions are key!

- Focus on peer instruction, using classroom voting to promote student discussions about key mathematical concepts.
- Learning to “talk math” means learning to “think math.” Students learn to logically defend mathematical ideas!
- No points or penalties for right or wrong answers. The primary purpose of the questions is to teach, not to assess and evaluate.

Making Connections

What does it mean if 0 is an eigenvalue of a matrix A ?

- (a) The determinant of A is zero.
- (b) The columns of A are linearly dependent.
- (c) There are an infinite number of solutions to the system $Ax = 0$.
- (d) All of the above
- (e) None of the above

Theory

The set of all 2x2 matrices with determinant equal to zero is not a vector space. Why?

(a) 2 x 2 matrices are not vectors.

(b) With matrices, AB need not equal BA.

(c) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 2 & 2 \end{bmatrix}$ and $\begin{bmatrix} 2 & 3 \\ 2 & 2 \end{bmatrix}$ is not in the set.

(d) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ is not in the set.

(e) None of the above

Lead-in to Linear Combinations

If $u = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$ and $v = \begin{bmatrix} -2 \\ 0 \\ 5 \end{bmatrix}$, what is $2u - 3v$?

a. $\begin{bmatrix} -4 \\ 4 \\ 23 \end{bmatrix}$

b. $\begin{bmatrix} 8 \\ 4 \\ -7 \end{bmatrix}$

c. $\begin{bmatrix} 8 \\ 4 \\ 23 \end{bmatrix}$

d. $\begin{bmatrix} 7 \\ 6 \\ 2 \end{bmatrix}$

Follow-up

Write $z = \begin{bmatrix} -5 \\ 3 \\ 16 \end{bmatrix}$ as a linear combination of $x = \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$

and $y = \begin{bmatrix} -3 \\ 2 \\ 6 \end{bmatrix}$.

a. $z = -5x$

b. $z = -2x + y$

c. $z = x + 2y$

d. $z = 2x + 2y$

e. z cannot be written as a linear combination of x and y .

f. None of the above

Graphical Follow-up

Write the vector w as a linear combination of u and v .

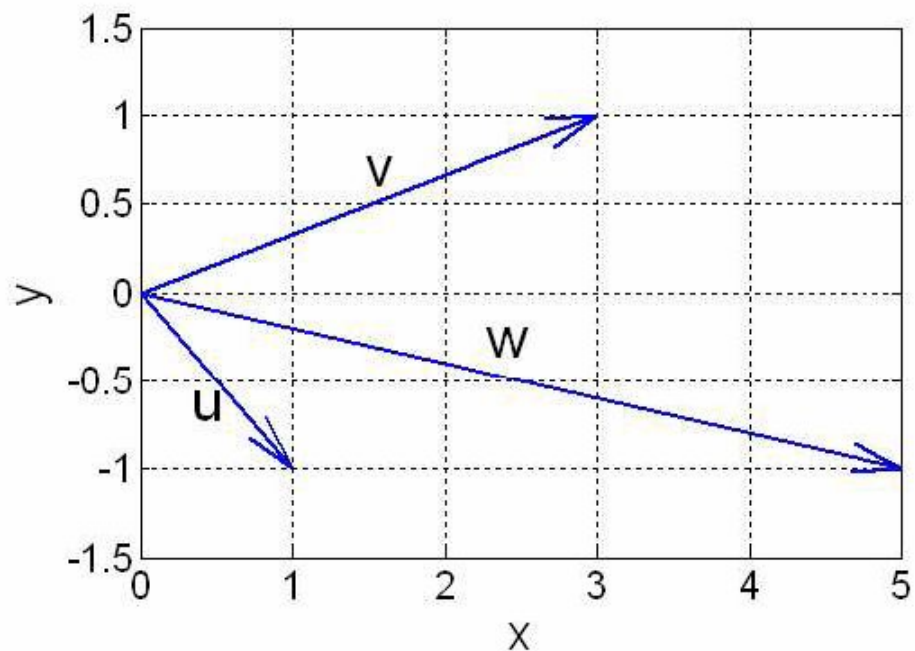
(a) $w = 2u + v$

(b) $w = u + v$

(c) $w = -u + v$

(d) $w = u - v$

(e) w cannot be written as a linear combination of u and v .



Application Follow-up

- Also extension and lead-in to nonnegative combinations

Howard's store sells three blends of our: standard, extra wheat, and extra soy. Each is a blend of whole wheat our and soy our, and the table below shows how many pounds of each type of our is needed to make one pound of each blend.

Standard Blend	Extra Wheat	Extra Soy	
0.5	0.8	0.3	Whole wheat flour
0.5	0.2	0.7	Soy flour

To save rent money, the store will be moving to a smaller space and will need to cut back on inventory. If possible, the manager would like to only stock two of these blends, and make the third from those as necessary. Which blends can be made from the others?

- (a) Standard Blend can be made from Extra Wheat and Extra Soy.
- (b) Extra Wheat can be made from Standard Blend and Extra Soy.
- (c) Extra Soy can be made from Standard Blend and Extra Wheat.
- (d) Any one blend can be made from the other two.

Timing Matters

A system of 5 linear equations and 7 variables could not have exactly _____ solutions.

(a) 0

(b) 1

(c) Infinite

(d) More than one of these is impossible.

(e) All of these are possible numbers of solutions.

Math QUEST

- NSF-funded project at Carroll College (DUE-0536077)
 - Zullo, Cline, and Parker
- Write questions for linear algebra and differential equations courses
- Need testers
- mathquest.carroll.edu